Pergamon

Int. J. Heat Mass Transfer. Vol. 41, No. 1, pp. 197–202, 1998

Printed in Great Britain
0017-9310/98 $19.00+0.00

PII: S0017-9310(97)00121-X

Non-linear mass transfer and Marangoni effect in gas–liquid systems

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(*Received* 15 *November* 1996)

Abstract—The comparative analysis of the influence of the effect of non-linear mass transfer and the Marangoni effect on the mass transfer kinetics and the hydrodynamic stability are done. Co-current gas and liquid flow in the laminar boundary layer along the flat phase surface is considered. One of the components of gas is absorbed by the liquid and reacts with a liquid component. The chemical reaction rate is of first-order. The heat effect from the chemical reaction creates a temperature gradient, i.e. the mass transfer together with a heat transfer can be observed. The data for heat and mass transfer coefficients are obtained. The critical Reynolds numbers, corresponding wave velocities, and wave numbers are obtained.

INTRODUCTION

Intensification of mass transfer in the industrial gas–liquid systems is obtained very often by the creation of large concentration gradients. It can be reached in a number of cases as a result of a chemical reaction of the transferred substance in the liquid phase. The heat effect of the chemical reactions creates significantly high temperature gradients. The temperature and concentration gradients created can considerably affect the mass transfer kinetics in gas–liquid systems. Hence, the experimentally defined mass transfer coefficients are significantly different from the ones predicted by the linear theory of mass transfer.

It has been shown in a number of papers [1–7], that the temperature, and the concentration gradients directly tangentially to the interface, can create a interfacial tension gradient. As a result of this a secondary flow is induced. The rate of the induced flow is directed tangentially to the interface. It leads to a change in the velocity distribution in the boundary layer and therefore, to a change in the mass transfer kinetics and hydrodynamic stability of the flow. These effects are considered to be the Marangoni effect and used as an explanation of all experimental deviations from the linear theory of the mass transfer, where the hydrodynamic of the flow does not depend on the mass transfer.

The investigations of gas–liquid systems with intensive interphase mass transfer as a result of large concentration gradients show [8–24] that under these conditions the secondary flow is induced normally directed to the surface. It leads to 'injection' or 'suction' of a substance in the boundary layer, therefore, to a change in the velocity distribution in the layer and the mass transfer kinetics and hydrodynamic stability of the flow. This effect of non-linear mass transfer can explain a number of experimental deviations from the linear theory of mass transfer [21], which have been explained with the Marangoni effect.

The two above-mentioned effects (the Marangoni effect and the effect of non-linear mass transfer) can manifest themselves separately, as well as in a combined effect. That is why their influence on the mass transfer kinetics and the hydrodynamic stability should be compared.

MATHEMATICAL MODEL

Co-current gas and liquid flow in the laminar boundary layer along the flat phase surface will be considered. One of the components of gas is absorbed by the liquid and reacts with a liquid component. The chemical reaction rate is of the first-order. The heat effect from the chemical reaction creates a temperature gradient, i.e. the mass transfer, together with a heat transfer, can be observed. Under these conditions the mathematical model takes the following form:

$$u_j\frac{\partial u_j}{\partial x}+v_j\frac{\partial u_j}{\partial y}=\nu_j\frac{\partial^2 u_j}{\partial y^2}\quad \frac{\partial u_j}{\partial x}+\frac{\partial v_j}{\partial y}=0$$

$$u_j\frac{\partial c_j}{\partial x}+v_j\frac{\partial c_j}{\partial y}=D_j\frac{\partial^2 c_j}{\partial y^2}-(j-1)kc_j$$

$$u_j\frac{\partial t_j}{\partial x}+v_j\frac{\partial t_j}{\partial y}=a_j\frac{\partial^2 t_j}{\partial y^2}+(j-1)\frac{q}{\rho_j c_{pj}}kc_j;$$

$$j=1\text{—gas},\quad i=2\text{—liquid}. \tag{1}$$

The influence of the temperature on the chemical reaction rate is not considered in equation (1) because it does not have a considerable effect on the comparative analysis of these two effects, which are the subject of the present investigation.

NOMENCLATURE

A	dimensionless wavenumber	β_r	the circle frequency
a	thermal diffusivity [$m^2\ s^{-1}$]	β_i	increment factor
C	dimensionless phase velocity	β/α	phase velocity
c	concentration [$kg\ mol\ m^{-3}$]	π	3.14
c_p	specific heat [$J\ kg^{-1}\ deg^{-1}$]	λ	heat-conductivity coefficient [$J.s\ m^{-2}$], wavelength [m]
D	diffusivity [$m^2\ s^{-1}$]		
i	imaginary unit	μ	dynamic viscosity [$N\ (m.s.\ deg)^{-1}$]
k	chemical reaction rate [$m\ s^{-1}$]	ρ	density [$kg\ m^{-3}$]
k_c	mass transfer coefficient [$m\ s^{-1}$]	σ	interfacial tension [$N\ m^{-1}$]
k_t	heat transfer coefficient [$m\ s^{-1}$]	χ	Henry constant.
M	molecule mass [$kg\ kg^{-1}\ mol^{-1}$]		
q	heat effect of the chemical reaction [$J\ kg^{-1}\ mol^{-1}$]		
		Indices	
t	temperature [deg], time	0	for a boundary value
u	velocity in x-direction [$m\ s^{-1}$]	1	for gas
v	velocity y-direction [$m\ s^{-1}$]	2	for liquid
x	coordinate [m]	*	for interface
y	coordinate [m].	r	for the real part of a complex number
Greek symbols		i	for the imaginary part of a complex number.
α	wave number [m^{-1}]		

The boundary conditions of equation (1) determine the potential gas and liquid flows far from the phase boundary. Thermodynamic equilibrium and the continuity of velocity and the flux of momentum, mass and heat fluxes can be detected on the phase boundary. It has been shown in ref. [18] that in the gas–liquid systems the effect of non-linear mass transfer is located in the gas phase. Taking into account these considerations, the boundary conditions take the following form:

$$x = 0 \quad u_j = u_{j0} \quad c_1 = c_{10} \quad c_2 = 0 \quad t_j = t_0$$

$$y \to \infty \quad u_1 = u_{10} \quad c_1 = c_{10} \quad t_1 = t_0$$

$$y \to -\infty \quad u_2 = u_{20} \quad c_2 = 0 \quad t_2 = t_0$$

$$y = 0 \quad u_1 = u_2 \quad \mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y} - \frac{\partial \sigma}{\partial x}$$

$$v_1 = -\frac{MD_1}{\rho_{10}^*} \frac{\partial c_1}{\partial y} \quad v_2 = 0$$

$$c_1 = \chi c_2 \quad D_1 \frac{\rho_1^*}{\rho_{10}^*} \frac{\partial c_1}{\partial y} = D_2 \frac{\partial c_2}{\partial y} \quad (\rho_1^* = \rho_{10}^* + Mc_1^*)$$

$$t_1 = t_2 \quad \lambda_1 \frac{\partial t_1}{\partial y} + \rho_1 c_{p1} v_1 t_1 = \lambda_2 \frac{\partial t_2}{\partial y} \quad j = 1, 2. \tag{2}$$

At a high enough value of c_0, one can observe a large concentration gradient normally directed to the interface $(\partial c_1/\partial y)_{y=0}$, which induces a secondary flow with the rate v_1. The tangential concentration and temperature gradients on the phase boundary create the interfacial tension gradient:

$$\frac{\partial \sigma}{\partial x} = \frac{\partial \sigma}{\partial c_2} \frac{\partial c_2}{\partial x} + \frac{\partial \sigma}{\partial t_2} \frac{\partial t_2}{\partial x} \tag{3}$$

which induces a tangential secondary flow, whose rate is proportional to $\partial \sigma/\partial x$. Further, we will examine the substance which are not surface active, i.e. $\partial \sigma/\partial c_2 \approx 0$.

HEAT AND MASS TRANSFER KINETICS

The mass transfer rate (J_c) and the heat transfer rate (J_c) can be determined from the local mass (I_C) and heat (I_t) fluxes after taking the mean of these fluxes along a length (l) of the interface:

$$J_c = k_c c_0 = \frac{1}{l} \int_0^l I_c \, dx$$

$$I_c = \frac{MD_1 \rho_1^*}{\rho_{10}^*} \left(\frac{\partial c_1}{\partial y} \right)_{y=0}$$

$$J_t = k_t t_0 = \frac{1}{l} \int_0^l I_t \, dx$$

$$I_t = -\lambda_1 \left(\frac{\partial t_1}{\partial y} \right)_{y=0} + \rho_1 c_{p1} (v_1 t_1)_{y=0} \tag{4}$$

where c_1 and t_1 are determined after solving the problems (1)–(2). In order to do this the following dimensionless variables will be introduced:

$$x = lX \quad y = (-1)^{j+1} \delta_j Y_j \quad \delta_j = \sqrt{\frac{v_j l}{u_{j0}}}$$

$$u_j = u_{j0} U_j(X, Y_j)$$

$$v_j = (-1)^{j+1} u_{j0} \frac{\delta_j}{l} V_j(X, Y_j)$$

$$c_j = (-\chi)^{1-j} c_0 C_j(X, Y_j)$$

$$t_j = t_0 + (-1)^{j+1} t_0 T_j(X, Y_j) \quad j = 1, 2. \tag{5}$$

Introducing equation (5) into equations (1) and (2) leads to the following equations:

$$U_j \frac{\partial U_j}{\partial X} + V_j \frac{\partial U_j}{\partial Y_j} = \frac{\partial^2 U_j}{\partial Y_j^2}, \quad \frac{\partial U_j}{\partial X} + \frac{\partial V_j}{\partial Y_j} = 0$$

$$U_j \frac{\partial C_j}{\partial X} + V_j \frac{\partial C_j}{\partial Y_j} = \frac{1}{Sc_j} \frac{\partial^2 C_j}{\partial Y_j^2} - (j-1) Da C_j$$

$$U_j \frac{\partial T_j}{\partial X} + V_j \frac{\partial T_j}{\partial Y_j} = \frac{1}{Pr_j} \frac{\partial^2 T_j}{\partial Y_j^2} + (j-1) Q Da C_j \quad j = 1, 2 \tag{6}$$

where

$$Da = \frac{kl}{u_{20}} \quad Q = \frac{q c_0}{\chi \rho_2 c_{p2} t_0} \quad Sc_j = \frac{v_j}{D_j}$$

$$Pr_j = \frac{v_j}{a_j} \quad j = 1, 2. \tag{7}$$

The boundary conditions of equations (6) have the following form:

$$X = 0 \quad U_j = 1 \quad C_1 = 1 \quad C_2 = 0 \quad T_j = 0 \quad j = 1, 2$$

$$Y_1 \to \infty \quad U_1 = 1 \quad C_1 = 1 \quad T_1 = 0$$

$$Y_2 \to \infty \quad U_2 = 1 \quad C_2 = 0 \quad T_2 = 0$$

$$Y_1 = Y_2 = 0 \quad U_1 = \theta_1 U_2 \quad \theta_2 \frac{\partial U_1}{\partial Y_1} = -\frac{\partial U_2}{\partial Y_2} + \theta_4 \frac{\partial T_2}{\partial X}$$

$$V_1 = -\theta_3 \frac{\partial C_1}{\partial Y_1} \quad V_2 = 0 \quad C_1 + C_2 = 0 \quad T_1 + T_2 = 0$$

$$\theta_5 \frac{\partial C_1}{\partial Y_1} = \frac{\partial C_2}{\partial Y_2} \quad \theta_6 \frac{\partial T_1}{\partial Y_1} = \frac{\partial T_2}{\partial Y_2}. \tag{8}$$

The effect of convective transfer in equation (8) is omitted as a small of higher-order, while parameters θ have the following form:

$$\theta_1 = \frac{u_{20}}{u_{10}} \quad \theta_2 = \frac{\mu_1}{\mu_2} \sqrt{\frac{v_2}{v_1}} \left(\frac{u_{20}}{u_{10}}\right)^{3/2}$$

$$\theta_4 = \frac{\partial \sigma}{\partial t_2} \frac{t_0}{u_{20} u_2} \sqrt{\frac{v_2}{u_{20} l}} \quad \theta_3 = \frac{M c_0}{\rho_{10}^* Sc_1} \quad (\theta_4 < 0)$$

$$\theta_5 = \chi \frac{D_1}{D_2} \frac{\rho_1^*}{\rho_{10}^*} \sqrt{\frac{u_{10} v_2}{u_{20} v_1}} \quad \theta_6 = \frac{\lambda_1}{\lambda_2} \sqrt{\frac{u_{10} v_2}{u_{20} v_1}}. \tag{9}$$

From equations (4) and (5) we can obtain directly the expressions for the Sherwood and Nuselt numbers:

$$Sh = \frac{k_c l}{D_1} = M\sqrt{Re_1} \int_0^1 (1 + \theta_3 Sc_1 C_1^*) \left(\frac{\partial C_1}{\partial Y_1}\right)_{Y_1 = 0} dX$$

$$Nu = \frac{k_t l}{\lambda_1} = -Re_1 \left[\int_0^1 \left(\frac{\partial T_1}{\partial Y_1}\right)_{Y_1 = 0} dX + \theta_3 Pr_1 \int_0^1 (1 + T_1^*) \left(\frac{\partial C_1}{\partial Y_1}\right)_{Y_1 = 0} dX \right]$$

$$C_1^* = C_1(X, 0), T_1^* = T_1(X, 0)$$

$$Re_1 = \frac{u_{10} l}{v_1} \quad Pr_1 = \frac{u_{10} l}{D_1}. \tag{10}$$

NUMERICAL METHOD

The problem (6) with the appropriate set of boundary conditions can be solved conveniently using the following algorithm:

$$U_1^{(k)} \frac{\partial U_1^{(k)}}{\partial X} + V_1^{(k)} \frac{\partial U_1^{(k)}}{\partial Y_1} = \frac{\partial^2 U_1^{(k)}}{\partial Y_1^2} \quad \frac{\partial U_1^{(k)}}{\partial X} + \frac{\partial V_1^{(k)}}{\partial Y_1} = 0$$

$$X = 0 \quad U_1^{(k)} = 1$$

$$Y_1 = 0 \quad U_1^{(k)} = \theta_1 U_2^{(k-1)} \quad V_1^{(k)} = -\theta_3 \frac{\partial C_1^{(k-1)}}{\partial Y_1}$$

$$Y_1 \to \infty \quad (Y_1 \geqslant Y_{1\infty}) \quad U_1^{(k)} = 1$$

$$0 \leqslant X \leqslant 1 \quad 0 \leqslant Y_1 \leqslant Y_{1\infty} \quad \theta_1 = 0.1 \quad Y_{1\infty} = 6$$

$$\text{(at the first iteration } \theta_1 = \theta_3 = 0\text{)}. \tag{11}$$

$$U_2^{(k)} \frac{\partial U_2^{(k)}}{\partial X} + V_2^{(k)} \frac{\partial U_2^{(k)}}{\partial Y_2} = \frac{\partial^2 U_2^{(k)}}{\partial Y_2^2} \quad \frac{\partial U_2^{(k)}}{\partial X} + \frac{\partial V_2^{(k)}}{\partial Y_2} = 0$$

$$X = 0 \quad U_2^{(k)} = 1$$

$$Y_2 = 0 \quad \frac{\partial U_2^{(k)}}{\partial Y_2} = -\theta_2 \left(\frac{\partial U_1^{(k)}}{\partial Y_1}\right)_{Y_1 = 0} + \theta_4 \left(\frac{\partial T_2^{(k-1)}}{\partial X}\right)_{Y_2 = 0} \quad V_2^{(k)} = 0$$

$$Y_2 \to \infty \quad (Y_2 \geqslant Y_{2\infty}) \quad U_2^{(k)} = 1$$

$$0 \leqslant X \leqslant 1 \quad 0 \leqslant Y_2 \leqslant Y_{2\infty} \quad \theta_2 = 0.145 \quad Y_{2\infty} = 6$$

$$\text{(at the first iteration } \theta_4 = 0\text{)}. \tag{12}$$

$$U_1^{(k)} \frac{\partial C_1^{(k)}}{\partial X} + V_1 \frac{\partial C_1^{(k)}}{\partial Y_1} = \frac{1}{Sc_1} \frac{\partial C_1^{(k)}}{\partial Y_1^2}$$

$$X = 0 \quad C_1^{(k)} = 1$$

$$Y_1 = 0 \quad C_1^{(k)} = -C_2^{(k-1)}(X, 0)$$

$$Y_1 \to \infty (Y_1 \geqslant \bar{Y}_1) \quad C_1^{(k)} = 1$$

$$0 \leqslant X \leqslant 1 \quad 0 \leqslant Y_1 \leqslant \bar{Y}_1 \quad Sc_1 = 0.735 \quad \bar{Y}_1 = 7$$

$$\text{(at the first iteration } C_2^{(k)}(X, 0) = 0\text{)}. \tag{13}$$

$$U_2^{(k)}\frac{\partial C_2^{(k)}}{\partial X}+V_2^{(k)}\frac{\partial C_2^{(k)}}{\partial Y_2}=\frac{1}{Sc_2}\frac{\partial^2 C_2^{(k)}}{\partial Y_2^2}-DaC_2^{(k)}$$

$$X=0 \quad C_2^{(k)}=0$$

$$Y_2=0 \quad \frac{\partial C_2^{(k)}}{\partial Y_2}=\theta_5\left(\frac{\partial C_1^{(k)}}{\partial Y_1}\right)_{Y_1=0}$$

$$Y_2\to\infty(Y_2\geqslant \bar{Y}_2) \quad C_2^{(k)}=0$$

$$0\leqslant X\leqslant 1 \quad 0\leqslant Y_2\leqslant \bar{Y}_2$$

$$Sc_2=564 \quad \theta_5=18.3 \quad \bar{Y}_2=0.26 \quad Da=10. \tag{14}$$

$$U_1^{(k)}\frac{\partial T_1^{(k)}}{\partial X}+V_1\frac{\partial T_1^{(k)}}{\partial Y_1}=\frac{1}{Pr_1}\frac{\partial^2 T_1^{(k)}}{\partial Y_1^2}$$

$$X=0 \quad T_1^{(k)}=0$$

$$Y_1=0 \quad T_1^{(k)}=T_2^{(k-1)}(X,0)$$

$$Y_1\to\infty(\bar{\bar{Y}}_1\geqslant Y_1) \quad T_1^{(k)}=0$$

$$0\leqslant X\leqslant 1 \quad 0\leqslant Y_1\leqslant \bar{\bar{Y}}_1 \quad Pr_1=0.666 \quad \bar{\bar{Y}}_1=7.4$$

$$\text{(at the first iteration } T_2^{(k)}(X,0)=0). \tag{15}$$

$$U_2^{(k)}\frac{\partial T_2^{(k)}}{\partial X}+V_2^{(k)}\frac{\partial T_2^{(k)}}{\partial Y_2}=\frac{1}{Pr_2}\frac{\partial^2 T_2^{(k)}}{\partial Y_2^2}+QDaC_2^{(k)}$$

$$X=0 \quad T_2^{(k)}=0$$

$$Y_2=0 \quad \frac{\partial T_2^{(k)}}{\partial Y_2}=\theta_6\left(\frac{\partial T_1^{(k)}}{\partial Y_1}\right)_{Y_1=0}$$

$$Y_2\to\infty(Y_2\geqslant \bar{\bar{Y}}_2) \quad T_2^{(k)}=0$$

$$Pr_2=6.54 \quad \theta_6=0.034 \quad \bar{\bar{Y}}_2=2.4 \quad QDa=8.6. \tag{16}$$

Certain values of the parameters in equations (11)–(16), as well as those to the end of the research, are calculated for the process of absorption of NH_3 in water or water solutions of strong acids.

The solution of equations (11)–(16) allows the determination of

$$J_1=\int_0^1\left(\frac{\partial C_1}{\partial Y_1}\right)_{Y_1=0}\mathrm{d}X$$

$$J_2=\int_0^1 C_1(X,0)\left(\frac{\partial C_1}{\partial Y_1}\right)_{Y_1=0}\mathrm{d}X$$

$$J_3=\int_0^1\left(\frac{\partial T_1}{\partial Y_1}\right)_{Y_1=0}\mathrm{d}X$$

$$J_4=\int_0^1 T_1(X,0)\left(\frac{\partial C_1}{\partial Y_1}\right)_{Y_1=0}\mathrm{d}X. \tag{17}$$

The introduction of equation (17) into equation (10) allows the determination of the Sherwood and Nusselt numbers:

$$Sh=M\sqrt{Re_1}(J_1+\theta_3 Sc_1 J_2)$$

$$Nu=-\sqrt{Re_1}[J_3+\theta_3 Pr_1(J_1+J_4)]. \tag{18}$$

NUMERICAL RESULTS

The results obtained by solution of problems (11)–(16) are shown in Table 1. The comparison of values of J_k ($k=1,\ldots,4$) shows that the rate of mass and heat transfer depends sufficiently on concentration gradient (θ_3) and in the cases of absorption ($\theta_3=0.2$) and desorption ($\theta_3=-0.2$) the mass and heat transfer coefficients are different with about 10% from the linear theory predictions ($\theta_3=0$). The influence of interfacial tension gradient is insufficient under real conditions ($0<\theta_4<1$). The increase of this parameter does not affect the heat and mass transfer kinetics.

LINEAR STABILITY ANALYSIS

In a great number of papers [1–7, 25] it has been shown that the tangential flows (as a result of interfacial tension gradients) have a considerable effect on the hydrodynamic stability upon the interface and the flow in the boundary layer. The induction of normal flows (due to large concentration gradients) have an effect of 'injection' or 'suction' of fluid in the boundary layer, which also changes the hydrodynamic stability in the boundary layer [18, 26–29]. It has been shown

Table 1. Values of average mass and heat fluxes ($Da=10$, $\theta_1=0.1$, $\theta_2=0.145$, $\theta_5=18.3$, $\theta_6=0.034$)

No.	θ_3	θ_4	J_1	J_2	J_3	J_4
1.	0	0	0.5671	0.09721	−0.01855	0.01337
2.	0.2	0	0.6129	0.11155	−0.02143	0.01554
3.	−0.2	0	0.5274	0.08542	−0.01623	0.01162
4.	0	10^{-4}	0.5671	0.09721	−0.01855	0.01338
5.	0	10^{-3}	0.5671	0.09721	−0.01855	0.01337
6.	0	10^{-2}	0.5670	0.09718	−0.01857	0.01339
7.	0	10^{-1}	0.5658	0.09696	−0.01879	0.01364
8.	0	1	0.5658	0.09696	−0.01879	0.01364

in ref. [28] that changes in the normal component of the velocity on the interface influence hydrodynamic stability stronger than changes in the tangential component.

The results obtained, solving equations (11)–(16), give the opportunity to define the influence of the nonlinear mass transfer and the Marangoni effect on the hydrodynamic stability of the flow in the boundary layer.

The linear analysis of hydrodynamic stability of the flows in the laminar boundary layers will be considered in the approximations of almost parallel flows. Flow with the velocity components $u(x,y)$ and $v(x,y)$ substituted with periodic disturbance with infinitesimal amplitude:

$$\bar{u} = G'(y)\exp i(\alpha x-\beta t) \quad \bar{v} = -i\alpha G(y)\exp i(\alpha x-\beta t) \tag{19}$$

where

$$\alpha = \frac{2\pi}{\lambda} \quad \beta = \beta_r + i\beta_i \quad \frac{\beta}{\alpha} = c_r - ic_i. \tag{20}$$

It can be seen directly from equations (19) and (20) that the disturbance amplitude will grow when $\beta_i > 0$ and damps when $\beta_i < 0$.

It has been demonstrated in ref. [26] that the amplitude of the periodic disturbance $G(y)$ satisfies the equation of Orr–Sommerfeld type:

$$\left(u-\frac{\beta}{\alpha}\right)(G''-\alpha^2 G) - \frac{\partial^2 u}{\partial y^2}G = -\frac{iv}{\alpha}(G^{iv}-2\alpha^2 G''+\alpha^4 G) + \frac{i}{\alpha}\left[vG''' - \left(\frac{\partial^2 v}{\partial y^2}-\alpha^2 v\right)G'\right]$$

$$y=0 \quad G=0 \quad G'=0$$

$$y\to\infty \quad G=0 \quad G'=0. \tag{21}$$

The analysis of the flow stability can be done directly if dimensionless variables are introduced into equation (21):

$$(\bar{U}_j - C_j)(\gamma_j'' - A_j^2\gamma_j) - \overline{\frac{\partial^2 U_j}{\partial Y_j^2}}\gamma_j = -\frac{i}{A_j Re_j}(\gamma_j^{iv} - 2A_j^2\gamma_j'' + A_j^4\gamma_j) + \frac{i}{A_j Re_j}\left(\bar{V}_j\gamma_j''' - \overline{\frac{\partial^2 V_j}{\partial Y_j^2}}\gamma_j' - A_j^2\bar{V}_j\gamma_j'\right) \quad j=1,2 \tag{22}$$

where

$$C_j = \frac{\beta_j}{\alpha_j u_{j0}} \quad A_j = \alpha_j\delta_j \quad Re_j = \frac{u_{j0}\delta_j}{v_j} = \sqrt{\frac{u_{j0}l}{v_j}}$$

$$\bar{U}_j = U_j(1,Y) \quad \bar{V}_j = V_j(1,Y_j) \quad \overline{\frac{\partial^2 U_j}{\partial Y_j^2}} = \left(\frac{\partial^2 U_j}{\partial Y_j^2}\right)_{X=1}$$

$$\overline{\frac{\partial^2 V_j}{\partial Y_j^2}} = \left(\frac{\partial^2 V_j}{\partial Y_j^2}\right)_{X=1} \quad \gamma_j(Y_j) = G(y) \quad j=1,2. \tag{23}$$

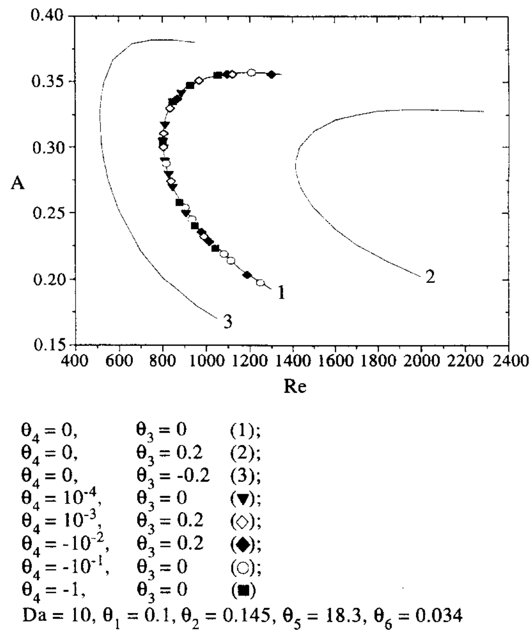

Fig. 1. The neutral curves of stability ($Re_{cr}A_0$) for flows of gases in the laminar boundary layer ($Da = 10$, $\theta_1 = 0.1$, $\theta_2 = 0.145$, $\theta_5 = 18.3$, $\theta_6 = 0.034$).

CRITICAL REYNOLDS NUMBERS

The solution of equation (22) has been found as it was done in refs [26–29], as a result the neutral curves of stability have been obtained (Fig. 1). The critical Reynolds numbers Re_{cr}, corresponding wave velocities C_r, and wave numbers A are obtained. C_{rmin} and A_{min} are also obtained from these results. We denote C_{rmin} and A_{min} the minimal values for wave velocities and wave number at which the flow is stable at any Reynolds number Re, respectively. They are shown in Table 2 for different values of the parameters

Table 2. Values of the critical Reynolds numbers Re_{cr}, corresponding wave velocities C_r, wave numbers A and C_{rmin}, A_{min} ($Da = 10$, $\theta_1 = 0.1$, $\theta_2 = 0.145$, $\theta_5 = 18.3$, $\theta_6 = 0.034$)

No.	θ_3	θ_4	Re_{cr}	A_{max}	C_{rmin}
1.	0	0	800	0.357	0.4503
2.	0.2	0	1411	0.329	0.4187
3.	−0.2	0	512	0.382	0.4763
4.	0	10^{-4}	800	0.357	0.4503
5.	0	10^{-3}	800	0.357	0.4503
6.	0	10^{-2}	800	0.357	0.4503
7.	0	10^{-1}	799	0.356	0.4505
8.	0	1	799	0.356	0.4505

θ_3, θ_4, taking into account the intensity of the secondary flows, as a result of concentration gradients and tangential temperature gradients.

RESULTS AND DISCUSSION

The numerical analysis of the influence of the effect on non-linear mass transfer and the Marangoni effect on the mass transfer kinetics and the hydrodynamic stability in gas–liquid systems leads to some basic conclusions:

(1) In the cases of absorption, the increase in intensity of the mass transfer directed from the volume of the gas phase toward the phase boundary ($\theta_3 > 0$) leads to an increase of the mass transfer rate (J_1) and an increase of the critical Reynolds numbers (Re_{cr}), i.e. stabilize the flow.
(2) In the cases of desorption the increase in intensity of the mass transfer directed from the phase boundary toward the volume of the gas phase ($\theta_3 < 0$) leads to a decrease of the mass transfer rate (J_1) and a decrease of the critical Reynolds numbers (Re_{cr}), i.e. destabilize the flow.
(3) The rise of the temperature gradient along the phase boundary length (θ_4) leads to a decrease of the mass transfer rate (J_1) and a decrease of the critical Reynolds numbers (Re_{cr}), i.e. destabilize the flow. This Marangoni effect, however, is insufficient in gas–liquid systems with movable phase boundary.
(4) The flow in the liquid phase is globally stable.

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